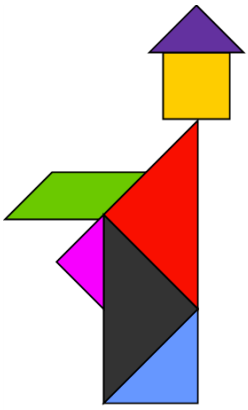
Thinking about our relationship with math teaching…

- Write down something in your math program that is difficult-to-learn, difficult-to-teach/ something you are interested in
  - An area that your students are struggling with and/or that you are curious about
- Most PD, least PD
- Highest comfort, lowest comfort

A Spatial Approach to Young Children's Mathematics



Tara Flynn & Sarah Bennett (TRENT)
Zack Hawes (OISE)

OVERVIEW of Partnerships

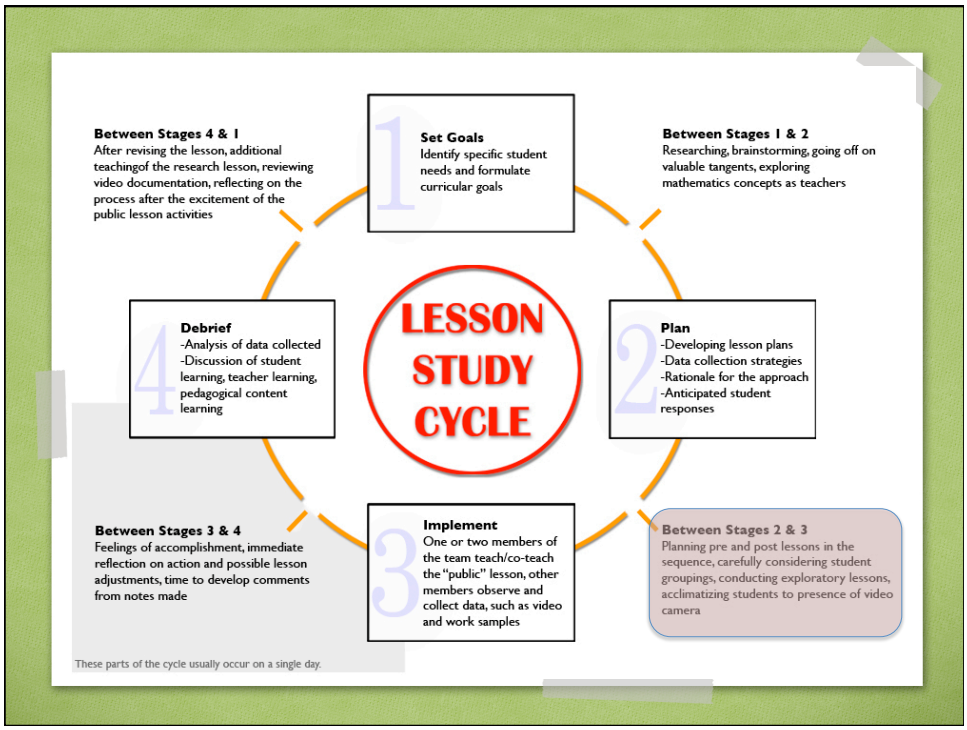
TRENT UNIVERSITY
TRENT MATHEMATICS EDUCATION RESEARCH COLLABORATIVE

OISE UT
Dr. Eric Jackman Institute of Child Study
UNIVERSITY OF TORONTO

Ontario
MINISTRY OF EDUCATION

Social Sciences and Humanities Research Council of Canada
Conseil de recherches en sciences humaines du Canada
Canada

ONTARIO SCHOOL BOARDS: KPRDSB, SMCDSB, TDSB, TCDSB, WCDSB
INDEPENDENT SCHOOLS: BSS, ICS



Layers of lesson study research:

Students
- Inquiring into mathematics (playing with math ideas)
- Helping us understand how students learn math (revealing both misconceptions and deep knowledge)

Teachers
- Investigating a specific math content area
- Engaging in lesson study
- Collecting evidence of student learning
- Developing dynamic learning kits
- LEARNING WITH RESEARCHERS

Researchers
- Facilitating lesson study
- Facilitating math content focus
- Collecting data
- LEARNING WITH TEACHERS

Topics

- Geometry: visualizing, verbalizing and verifying with shape arrangements and orientation
- Number Sense: composing and decomposing numbers
- Relative quantity and magnitude (number lines)
- Linear growing patterns
- Composing and decomposing two dimensional figures
- Three dimensional geometry and block play
- **Measurement: linear, area and perimeter**

What is spatial reasoning?

The ability to create and manipulate mental representations of actual and imagined shapes, objects, and structures (Cohen & Hegarty, 2012).

Spatial reasoning is *not* a unitary construct and involves many related and unrelated skills, including:

navigating and wayfinding manipulating objects

crafing and reading maps, graphs, visual data visualizing

locating objects and remembering locations of objects

imagining objects move in space (mental rotation and transformations)

creating or designing objects perspective taking

imagining objects moving in space scaling up or down

moving one's body in space proportional reasoning

non-verbal reasoning comparing shifting dimension

disembedding orienting diagramming

composing and decomposing

Pure spatial abilities

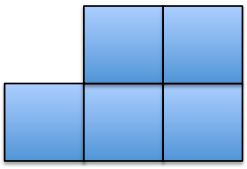
- not embedded in mathematical context and not used to represent quantity BUT proven strongly linked to math skills
  - Visuospatial working memory
  - Mental rotation
  - Perspective-taking

Top, Front & Side Views

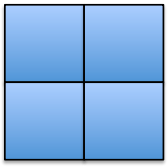
set 1

Try this: VISUALIZE, VERBALIZE, VERIFY!

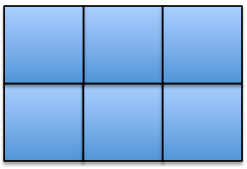
SIDE VIEW



FRONT VIEW



TOP VIEW

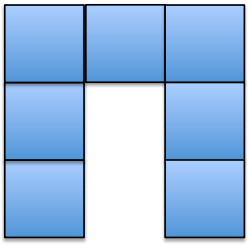


Top, Front & Side Views

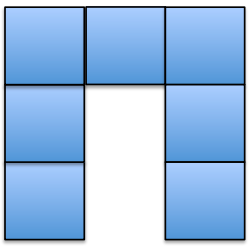
set 2

Try this: VISUALIZE, VERBALIZE, VERIFY!

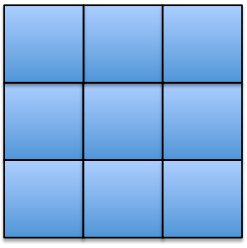
SIDE VIEW



FRONT VIEW

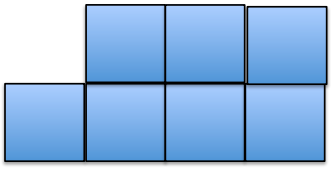


TOP VIEW

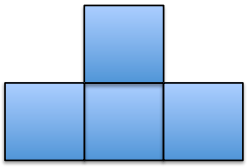


Top, Front & Side Views
set 3

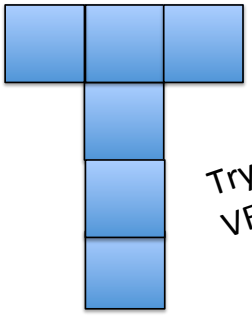
FRONT VIEW



SIDE VIEW



TOP VIEW



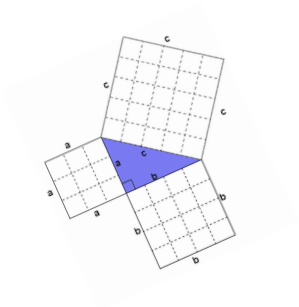
Try this: VISUALIZE, VERBALIZE, VERIFY!

Notice….

- What mathematics are students doing?
- How is this related to other math that students are doing?
- How do these kinds of activities relate to *number*?

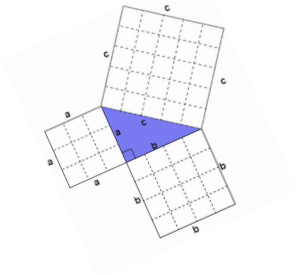
Visual Representations in Math

- Mathematicians have long been aware of the value of diagrams, models and other visual tools for teaching, and for developing mathematical thinking



However…

- Despite the obvious importance of visual images in human cognitive activities, visual representation remains a second class citizen in the teaching and learning of mathematics



Spatial Thinking & Mathematics: Why Care?

3 reasons why we should care about spatial thinking in our math programs:

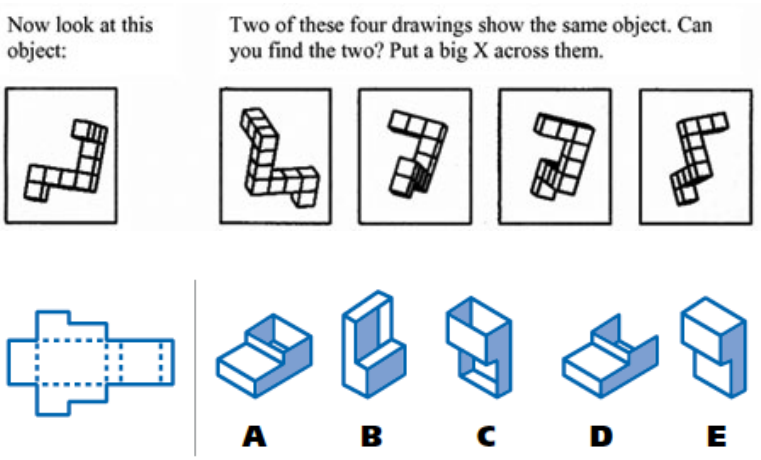
1) Spatial thinking and mathematical thinking are strongly related.

2) We need to pay more explicit attention to spatial thinking in schools.

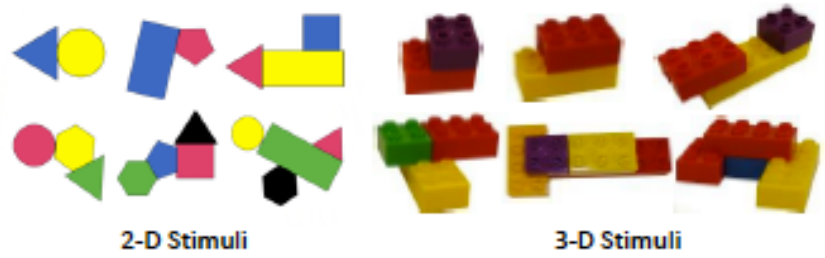
3) Spatial thinking can be improved – education matters!

1. Spatial thinking and mathematical thinking are strongly related

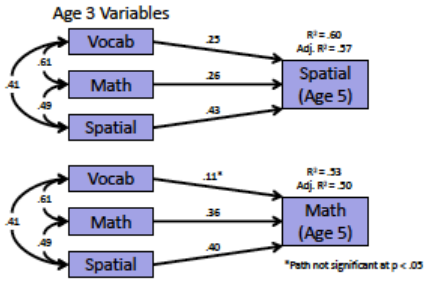
- People who perform well on measures of spatial reasoning also tend to perform well on measures of mathematics (Mix & Cheng, 2012).



1. Spatial thinking and mathematical thinking are strongly related



2-D Stimuli 3-D Stimuli

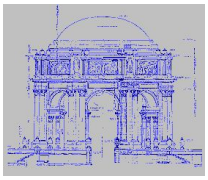


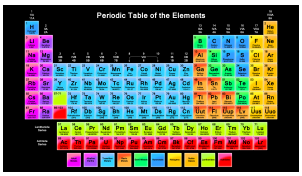
Spatial performance at age 3 predicted math performance at age 5 – even better than the math measure at age 3

Farmer et al., 2013

1. Spatial thinking and mathematical thinking are strongly related

Examples of spatial activities that relate to math performance:

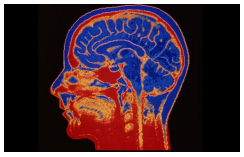




- **Quality of block play shown to predict performance in high school mathematics** (Wolfgang, Stannard, & Jones, 2001).

- **Entrance and success in STEM disciplines related to high school spatial reasoning abilities (**(Wai, Lubinksi, Benbow, 2009).

Further evidence for link between spatial thinking and mathematics come from studies showing overlapping brain regions involved in both activities (Geary, 2007).



- Empirical evidence indicates that spatial imagery reflects not just general intelligence *but also the ability to solve mathematical problems, especially non-routine problems* (e.g., Casey, Nutall & Pezaris, 1992, 1997, 2001; Wheatley et al. 1994)

A little story about a guy named Einstein…

- *Einstein discovered the law of relativity by visualizing himself riding a beam of light*
- *And one named Hawking, who because of his abilities is ONLY able to visualize: "By losing the finer dexterity of my hands, I was forced to travel the universe in my mind, and try to visualize the ways in which it worked."*

- Although HOW they are related is still unclear…and what spatial processes help with what mathematical abilities is still a huge question
  - multiple spatial abilities and multiple math abilities make a "complex matrix" (Mix & Cheng, 2012)

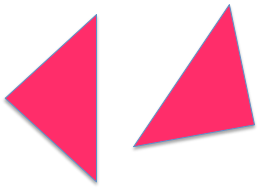
2. We need to pay more explicit attention to spatial thinking.

- Geometry and spatial sense receive the *least* amount of attention in early years math (M4YC, 2012; Sarama & Clements, 2009)

- When taught, focus is on labeling and sorting shapes

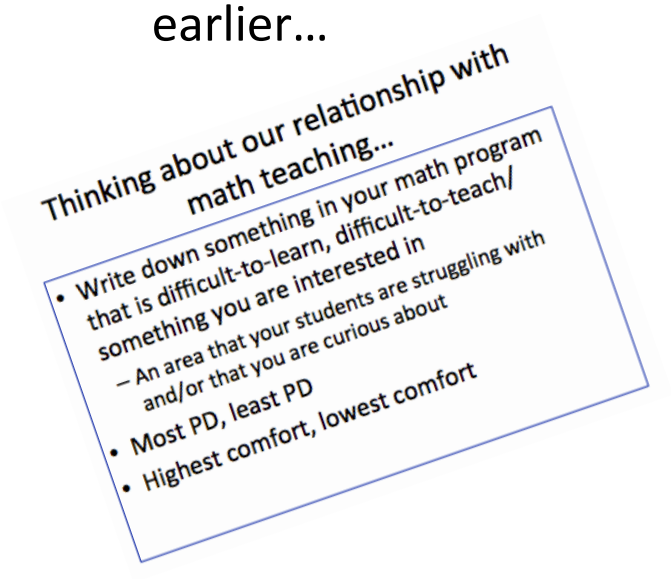
Can you make a square using only two triangles?

Will any two triangles work? What do you notice about the triangles that do go together to makes a square?



- This 'neglect' occurs despite calls (NCTM, 2006, 2010) to push geometry and spatial thinking to the forefront of early math curricula…

Think back to what you identified earlier…



A tiny bit of history on young children and geometry

- Froebel (1850's) "grandfather" of kindergarten
  - Emphasis on geometry and spatial sense
  - First 6 gifts: balls of different colours, cubes, spheres, cylinders, and a carefully constructed set of geometric blocks (progressive tasks)
- What remains of Froebel's early focus on mathematics in kindergarten is "largely forgotten or diluted" (Sarama & Clements, 2008)
- Focus on naming and sorting shapes, and little else (Beaton, 1996; Copley, 2000)

- Some individuals are harmed in their progression in mathematics due to lack of attention to spatial reasoning and geometry (e.g., Casey and Erkut 2005, Clements & Sarama, 2011)
- CONSIDER…how do we define ability in mathematics? Is it by number sense alone? Do we value students who may have emerging number sense but strong visuo-spatial ability???

3. Spatial thinking can be improved

A recent meta-analysis that looked at 217 studies found that spatial training (e.g., Tetris, puzzle play, block building, etc.) led to significant improvements on tasks trained but also on non-trained tasks (Uttal et al., 2012).

3. Spatial thinking can be improved

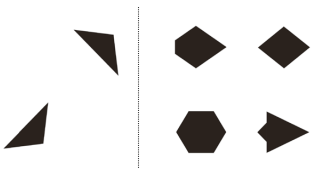
But…Do improvements in spatial thinking lead to improvements in math performance?

Study 1:
Pre-school children who took part in an after school program that focused on creating and copying intricate designs, demonstrated improvements in spatial reasoning, executive functions, and mathematics (including arithmetic based tasks) compared to a control group of children who attended the regular after school program (Grissmer, 2013).

3. Spatial thinking can be improved

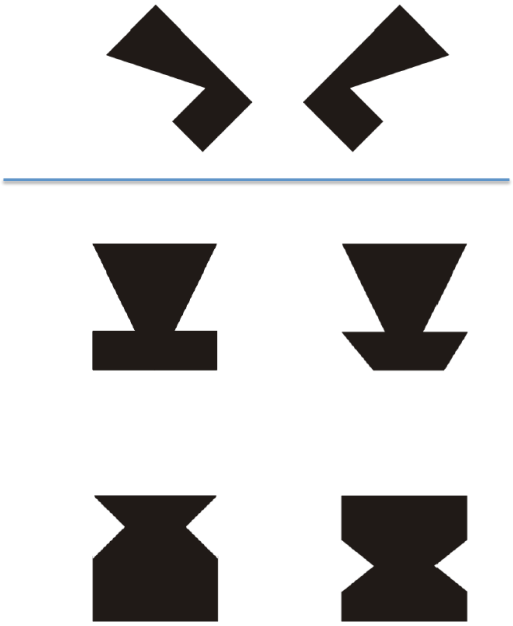
But…Do improvements in spatial thinking lead to improvements in math performance?

Study 2: Children aged 6-8 assigned to mental rotation training or crossword condition. Mental rotation training led to significant improvements in both spatial reasoning and calculation ability. Improvement was largely due to better performance on missing term problems.



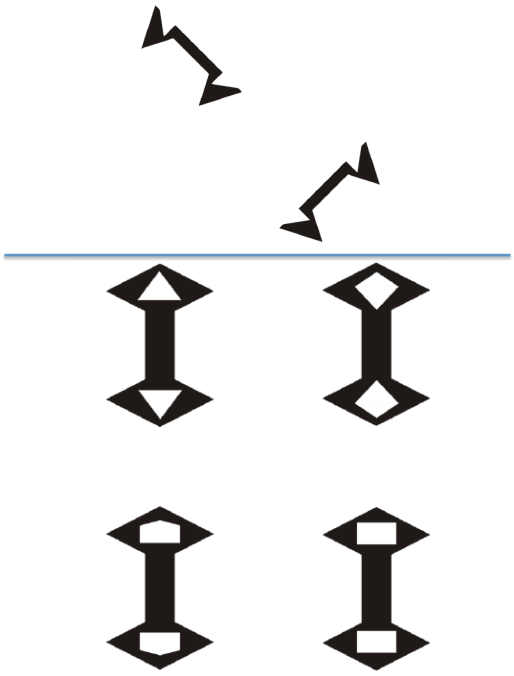
4 + 5 = _____
56 + 6 = _____
2 + _____ = 7

Levine Tasks



SK

Levine Tasks



JK

Other examples:

- Cheng & Mix (2011) found that 2D mental rotation task improved children's performance on math tasks (but not other spatial tasks)
- Uttal et al. (2012) found math improvements especially related to spatial perception tasks (e.g., task that requires people to draw a line where they think the water level would be in a series of tilted bottles)

Implications for the classroom:

You are already doing something about it!

Starting by understanding the importance of spatial thinking

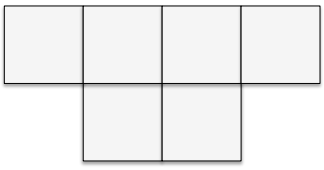
Implications for the classroom:

- Spatial reasoning is not an "add on" to an already loaded curriculum, but a process that lies across content areas and other subjects (e.g., art)
- "spatialize" the curriculum (Newcombe, 2013) in playful ways across content areas, and include:
  - Mental rotation tasks
  - Games like Tetris
  - Jig saw puzzles
  - Guided play with blocks and geometric shapes

"Playing with 6 squares"

Make 3 designs/figures using 6 square units

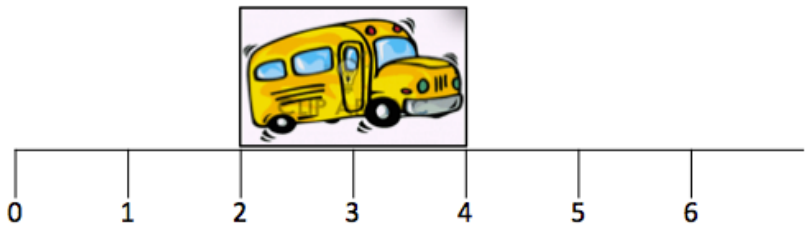
RULE: Edges of units must be flush (not corner to corner)



BIG IDEA: Shapes can have the same area even though they look different (fixed area but different shapes)

Implications for the classroom:

- Use of spatial language (see Newcombe, 2010) e.g., over, under, middle, parallel
- Importance of gestures (teacher and student) – use and encourage gestures
- Importance of visual displays of data e.g., graphs
- Emphasize geometry
- Importance of early years
- Importance of tools and technology



Strategy for promoting visualization: Visualize, verbalize, verify…

- Ask children to imagine and visualize as a first step?
- Promoting attention to visualization strategies: “What were your picturing in your head? (vs. “How do you know?”)

How does this apply to your learning context?

Stay tuned…

Upcoming research e.g., Research Forum at the Annual Conference for the

Psychology of Mathematics Education

(PME2014)

in Vancouver

www.tmerc.ca
tara.c.flynn@gmail.com

www.oise.utoronto.ca/robertson/
zack.hawes@utoronto.ca